Statistical Physics (PH312)
HW #5, Fall 2018

due : December 4, 2018

Note that you SHOULD show the details of your work.

1. In the class, we have derived the equation of state for the Bose gas up to the order of $O(n\lambda^3)^2$. Carry one term further to find the equation of state up to the next order.

2. For Bose-Einstein condensation, show that

$$\frac{C_v}{Nk_B} = \begin{cases} \frac{3}{2}\left(1 + \frac{n\lambda^3}{2^{7/2}} + \cdots\right) & (T > T_c) \\ \frac{15}{4}\frac{\zeta_{5/2}}{\zeta_{3/2}}\left(\frac{T}{T_c}\right)^{3/2} & (T < T_c). \end{cases}$$

3. (a) Draw the cluster diagrams corresponding to the following products of f-fucntions:

$$f_{12}f_{23}f_{34}f_{45}f_{51}f_{14}f_{25}$$
$$f_{12}f_{23}f_{13}f_{34}f_{45}f_{46}f_{56}$$

   (b) In the class, we derived the second and third virial coefficients, B_2 and B_3. Express the next virial coefficient B_4 in terms of the cluster integrals (or the cluster diagrams).
   (c) For a hard-sphere potential,

$$v(r) = \begin{cases} \infty & (r < a) \\ 0 & (a < r) \end{cases}$$

   calculate the third virial coefficient B_3.

4. (a) Consider a gas satisfying Dieterici's equation of state:

$$P(v-b) = k_B T \exp\left(-\frac{a}{k_B T v}\right)$$

   where $v = V/N$. Find the second virial coefficient $B_2(T)$ and also calculate $Pv/k_B T$ at the critical point.
   (b) Consider a gas consisting of particles interacting through a square-well potential as

$$v(r) = \begin{cases} \infty & (r < a) \\ -\epsilon & (a < r < b) \\ 0 & (b < r) \end{cases}$$

   Find the second virial coefficient $B_2(T)$.