Statistical Physics (PH312)
HW #4, Fall 2018

due : November 22, 2018

Note that you SHOULD show the details of your work.

1. In the class, we have derived the first order corrections for the degenerate Fermi gas (at low temperature and with high density). Carry one term further to find $\ln z$ up to the next order and find the expressions for the chemical potential μ, the pressure P, the helmholtz free energy F, the energy E, and the heat capacity C_v, up to the next order corrections.

2. Consider a white dwarf as a system of N electrons of mass m and $N/2$ helium nuclei of mass $4m_p$ (m_p is the proton mass). The star is also taken to be a sphere of radius R and mass M.
(a) The total mass is $M \approx 2Nm_p \approx 10^{30}$g at a density $\rho \approx 10^{10}$ Kg/m^{-3} and at a temperature $T \sim 10^7$K. Show that despite the large temperature the electron gas can be considered to highly degenerate but on the other hand relativistic effects are important.
(b) Considering the white dwarf as a degenerate Fermi gas composed of N relativistic electrons in its ground state, calculated the pressure exerted by the Fermi gas.
(c) Calculate the equilibrium radius of a star under the assumption that in equilibrium the pressure of the electrons just balances the gravitational pressure. Assume that the electron gas is uniformly distributed inside the star. Find the mass-radius relationship at nonrelativistic and relativistic limits, respectively.

3. (a) Derive Eq. (12.12) of K. Huang
$$P = \frac{1}{3}\frac{E}{V}$$
for a photon gas, and compare this to the analogous results for Bosons and Fermions with nonzero rest mass.
(b) For a photon gas, derive Planck's radiation law (Eq. (12.14) of K. Huang) and Stefan's radiation law (Eq. (12.18) of K. Huang).

4. For Bose-Einstein condensation, show that
$$\frac{\langle n_0 \rangle}{N} = \begin{cases} 0 & (T > T_c) \\ 1 - \left(\frac{T}{T_c}\right)^{3/2} & (T < T_c). \end{cases}$$
Also, express the critical temperature with the density n and the mass m.