Statistical Physics (PH312)
HW #2, Fall 2018

due : Oct 18, 2018

Note that you SHOULD show the details of your work.

1. Consider a random walk in one dimension. In a single step the probability of a displacement between x and $x + dx$ is given by
$$p(x)dx = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-a)^2}{2\sigma^2}\right).$$
After N steps, the displacement of the walker is $Y = x_1 + \cdots + x_N$ where x_i is the displacement in the ith step. Assume the steps are independent of one another.
(a) What is the probability distribution for the displacement Y?
(b) What are the average displacement, $\langle Y \rangle$ and the variance, $\langle (\Delta Y)^2 \rangle$?
(c) Repeat (a) and (b) for a random walk where the walker is equally likely to take a step with displacement in the interval $d - a \le x \le d + a$ for $a < d$.

2. Consider a monoatomic ideal gas consisting of N_0 molecules in a container of volume V_0. Show from the grand canonical ensemble picture that the number N of molecules contained in a region of volume V (anywhere inside the container) is given by the Poisson distribution, $P(N, V)$.
(a) Show at first that $\langle N \rangle = N_0 f$ and $\langle N^2 \rangle - \langle N \rangle^2 = N_0 f(1 - f)$ where $f = V/V_0$.
(b) Show that if both $N_0 f$ and $N_0(1 - f)$ are large numbers, $P(N, V)$ assumes a Gaussian distribution.
(c) Further, if $f \ll 1$ and $N \ll N_0$, show that
$$P(N, V) = \frac{\langle N \rangle^N}{N!} e^{-\langle N \rangle}.$$

3. (a) Determine the grand partition function for a gas system of magnetic atoms that can have, in addition to the kinetic energy, a magnetic potential energy $\mu_B H$ or $-\mu_B H$ (with $J = \frac{1}{2}$ and $g = 2$), depending on the orientation with respect to an applied magnetic field H.
(b) Derive the expression for the magnetization of the system, M_z.
(c) Calculate how much heat will be produced by the system when the magnetic field is reduced from H to zero at constant volume and constant temperature.

4. (a) For an isothermal-isobaric ensemble (NPT-ensemble), show that
$$P(V) = P(V^*) \exp\left(\frac{(V - \langle V \rangle)^2}{2kT(\partial V/\partial p)_{N,T}}\right).$$
where V^* is the most probable volume.
(b) For a grand canonical ensemble, show that
$$\langle (\Delta E)^2 \rangle = kT^2 C_V + \left(\frac{\partial U}{\partial N}\right)_{T,V}^2 \langle (\Delta N)^2 \rangle.$$
where $\langle (\Delta E)^2 \rangle \equiv \langle E^2 \rangle - \langle E \rangle^2$ and $\langle (\Delta N)^2 \rangle \equiv \langle N^2 \rangle - \langle N \rangle^2$.
(c) For a grand canonical ensemble, show that
$$\langle NE \rangle - \langle N \rangle \langle E \rangle = \left(\frac{\partial U}{\partial N}\right)_{T,V} \langle (\Delta N)^2 \rangle$$
and that
$$\langle (\Delta J)^2 \rangle = kT^2 C_V + \left\{\left(\frac{\partial U}{\partial N}\right)_{T,V} - \mu\right\}^2 \langle (\Delta N)^2 \rangle$$
where $J = E - N\mu$.