Nonequilibrium Statistical Mechanics

HW #1, Spring 2020

due : April 24, 2020

Note that you SHOULD show the details of your work.

1. (a) Consider the phase space probability density $f_N((\vec{p}, \vec{q}, t)$ for a classical system of N particles. Using the Hamilton's equation of motion, derive the Liouville equation that describes the time evolution of f_N.
(b) Derive the BBGKY hierarchy for the reduced distribution function f_s in the presence of external force $\mathbf{F}$.

2. (a) Consider a collision invariant $\chi(\vec{p}, \vec{q}, t)$ such that $\chi(\vec{p}) + \chi(\vec{p_1}) = \chi(\vec{p'}) + \chi(\vec{p'_1})$ during a collision. From the Boltzmann equation, derive the conservation equation for any collision invariant χ (Multiply the Boltzmann equation on both sides by χ and then integrate over $\vec{p}$).
(b) Letting $\chi = 1, p_\beta/m - u_\beta, (\vec{p} - m\vec{u})^2/2m$, derive the local balance equations for the particle number, the momentum, the kinetic energy.

3. The zeroth-order solution of the Chapman-Enskog expansion is assumed to be a local Maxwell distribution characterized by local density $n(\mathbf{r}, t)$, local mean velocity $\mathbf{u}(\mathbf{r}, t)$, and local temperature $\theta(\mathbf{r}, t) \equiv k_B T(\mathbf{r}, t)$:

$$f^{(0)}(\mathbf{r}, \mathbf{p}, t) = n(\mathbf{r}, t)\, [2\pi m\theta(\mathbf{r}, t)]^{-3/2} \exp\left(-\frac{m(\mathbf{v} - \mathbf{u}(\mathbf{r}, t))^2}{2\theta(\mathbf{r}, t)}\right).$$

(a) Using $f^{(0)}(\mathbf{r}, \mathbf{p}, t)$, evaluate the pressure tensor and the heat flux defined respectively as

$$P^{(0)}_{\alpha\beta} = nm\langle c_\alpha c_\beta \rangle, \quad \mathbf{h}^{(0)} = \frac{1}{2} nm^2 \langle \mathbf{c}\mathbf{c}^2 \rangle$$

where $\mathbf{c} = \mathbf{v} - \mathbf{u}$.
(b) Inserting these expressions for $P^{(0)}$ and $\mathbf{h}^{(0)}$ into conservation equations for the momentum and the energy, derive the zeroth-order hydrodynamic equations for a perfect fluid.

4. (a) The first-order solution can be obtained by making use of the relaxation time approximation. Show that

$$f^{(1)} \equiv f^{(0)}(1 + g) = f^{(0)} \left\{1 - \tau \left[\frac{1}{\theta}(\nabla\theta \cdot \mathbf{c}) \left(\frac{m}{2\theta}\mathbf{c}^2 - \frac{5}{2}\right) + \frac{1}{\theta}\mathbf{\Lambda} \cdot \left(\mathbf{c}\mathbf{c} - \frac{1}{3}\mathbf{c}^2 I\right)\right]\right\}.$$

where $\mathbf{\Lambda} = m\nabla\mathbf{u}$.
(b) Find the first-order pressure tensor $P^{(1)}_{ij}$ and the first-order heat flux $\mathbf{h}^{(1)}$. Also express the viscosity and the thermal conductivity in terms of local thermodynamic variables. Using the conservation equations, derive the first-order hydrodynamic equations.