Statistical Physics (PH312)

HW #5, Fall 2019

due : Dec 19, 2019

Note that you SHOULD show the details of your work.

1. Consider an three-dimensional ideal Bose gas.
   (a) Show that the pressure P is given by the Bose-Einstein function $g_n(z)$ as
   $$\beta P = \frac{g}{\lambda^3} g_{5/2}(z)$$
   and that the energy density is related to the pressure, $E/V = (3/2)P$. Here, g is the spin degeneracy factor, λ the thermal de Broglie wavelength, and z is the fugacity.
   (b) Now consider the non-degenerate case, i.e., high temperature T and low density n. Using perturbative expansion scheme, obtain the pressure in the form of the virial expansion up to the third order of the density, and calculate the heat capacity valid at high T regime, up to the leading correction to the classical result.

2. Consider a free Fermi gas with spin 1/2 in two dimensional space. Fermi gas is confined in a square with area $A = L^2$. Mass of particle is given as m.
   (a) Find the Fermi energy in terms of particle number N and area A. Also find the average energy of the particles.
   (b) Derive the formula of density of states. For a finite temperature T, express chemical potential μ as function of N and T.

3. Consider a one dimensional lattice of N identical point particles of mass m, interacting via nearest-neighbor spring-like forces with spring constant $m\omega^2$. Denote the lattice spacing by a. As is easily shown, the normal mode eigenfrequencies are given by
   $$\omega_n = \omega\sqrt{2(1 - \cos ka)} = \omega\sqrt{2(1 - \cos(2\pi n/N))},$$
   with $k = 2\pi n/aN$, and the integer n ranging from $-N/2$ to $N/2$ ($N \gg 1$). The system is in thermal equilibrium at temperature T. we want to find a behaviour of the specific heat(constant volume). Let C_v be the specific heat at a constant volume.
   (a) Compute C_v for the regime $T \to \infty$.
   (b) For $T \to 0$, we expect $C_v \to A\omega^{-\alpha}T^{\gamma}$ (where A is a constant that you need not compute). Compute the exponents α and γ.