Statistical Physics (PH312)
HW #3, Fall 2019

due : Oct 29, 2019

Note that you SHOULD show the details of your work.

1. Consider a system, in grand canonical ensemble, of a volume V, a temperature T, and a chemical potential μ.
(a) Show that
$$\langle(\Delta E)^2\rangle = \left\langle(\Delta E)^2\right\rangle_{can} + \left(\frac{\partial U}{\partial N}\right)^2_{T,V} \langle(\Delta N)^2\rangle.$$
where $\langle\cdots\rangle_{can}$ indicates the canonical ensemble average, while $\langle\cdots\rangle$ is the grand canonical ensemble average. $\langle(\Delta X)^2\rangle \equiv \langle X^2\rangle - \langle X\rangle^2$ is the variance of a random variable X.
(b) Show that
$$\langle NE\rangle - \langle N\rangle\langle E\rangle = \left(\frac{\partial U}{\partial N}\right)_{T,V} \langle(\Delta N)^2\rangle$$
(c) Show that the entropy S is given as
$$S = -\left(\frac{\partial \Phi}{\partial T}\right)_{\mu,V}$$
where the grand potential is $\Phi = -k_B T \ln Q$ with the grand partition function Q.

2. (a) Consider the ideal gas in grand canonical ensemble in volume V, temperature T and chemical potential μ. Derive the grand canonical partition function of the system.
(b) Show that for a system in grand canonical ensemble, the variance of particle number can be expressed as $\langle(\Delta N)^2\rangle \equiv \langle N^2\rangle - \langle N\rangle^2 = kT\frac{\partial\langle N\rangle}{\partial\mu}|_{T,V}$. Also, show that the variance of the particle number, $\langle(\Delta N)^2\rangle$ is given by $\langle N\rangle$.

3. Consider a classical system of *indistinguishable* noninteracting, diatomic molecules enclosed in a box of volume V at temperature T. The system is in grand canonical ensemble. The Hamiltonian of a single diatomic molecule is given by
$$H(\vec{r_1}, \vec{r_2}, \vec{p_1}, \vec{p_2}) = \frac{1}{2m}(p_1^2 + p_2^2) + \frac{1}{2}K|\vec{r_1} - \vec{r_2}|^2.$$
(a) Derive the grand canonical partition function in this system (use the approximation, $\int_V e^{-r^2} d^3r \approx \sqrt{\pi}$, for a large volume of the system).
(b) Calculate the grand potential Φ, then calculate the entropy S, the pressure P, the number of particle N from the grand potential.