Statistical Physics (PH312)

HW #1, Fall 2019

due : Oct 1, 2019

Note that you SHOULD show the details of your work.

1. Consider an isolated system of volume V including $N(\gg 1)$ particles. The Hamiltonian of the system is given by $\mathcal{H}(p, q)$ where p, q are vectors of the canonical momenta $p_1, p_2, \cdots, p_{3N}$ and the canonical coordinates $q_1, q_2, \cdots, q_{3N}$. Now, let's consider a microcanonical ensemble of the system which has an energy between E and $E + \Delta$(assume $\Delta \ll E$). We have three different ways to define an entropy for the ensemble

$$S_1 = k_B \log \Gamma(E) \tag{1}$$

$$S_2 = k_B \log \Sigma(E) \tag{2}$$

$$S_3 = k_B \log g(E) \tag{3}$$

where k_B is the Boltzmann's constant, $\Gamma(E)$ is the shell volume in the phase space occupied by the microcanonical ensemble

$$\Gamma(E) = \int_{E<\mathcal{H}(p,q)<E+\Delta} d^{3N}p \, d^{3N}q, \tag{4}$$

$\Sigma(E)$ is the volume in phase space enclosed by the energy surface of energy E

$$\Sigma(E) = \int_{\mathcal{H}(p,q)<E} d^{3N}p \, d^{3N}q, \tag{5}$$

and $g(E)$ is the density of states defined as

$$g(E) = \frac{\partial \Sigma(E)}{\partial E}. \tag{6}$$

(a) Show S_1 has the extensive property. More precisely, if a system is composed of two subsystems whose entropies are, respectively, S_a and S_b, the entropy of the total system is $S_a + S_b$, when the subsystems are sufficiently large.
(b) Show those three definitions are equivalent to one another upto negligible(order of $\log N$) difference.

2. Consider an isolated quantum mechanical system consisting of N non-interacting particles in a cubic box of length L. The state of the system is described by the $3N$ quantum numbers n_i ($i = 1, \cdots, 3N$) of the occupied states, and the total energy is given by

$$E = \frac{h^2}{8mL^2} \sum_{i=1}^{3N} n_i^2.$$

The number of microstates for given macroscopic quantities (E, V, N) is then given by the number of integer grid points satisfying the above relation. For a large $N(N \gg 1)$, the grid points are so dense that the average number of grid points up to the energy E can be assumed to be proportional to the volume of the positive "octant" ($n_i > 0$), up to E, spanned by $3N$ dimensional continuous coordiantes n_i. Note that the volume of the unit cube in (n_x, n_y, n_z)-space equals 1.
(a) Find the expression for the number of microstates $\Omega(E, V, N)$ and the entropy $S(E, V, N)$.
(b) Find the pressure P exerted by the particles on the box in terms of E, V, N.

3. Consider a two-dimensional classical ideal gas system in microcanonical ensemble where N particles are confined in area A with total energy E.
(a) Derive the entropy S in terms of the area A, total energy E and the particle number N.
(b) Derive the thermodynamic quantities, temperature T and pressure P.